(spontaneous $\rightarrow 10^{-8}$ sec)

Fermi-Golden Rule :-

or

Closely-Spaced Levels Rule :

or

Constant Perturbation :-

1. It gives the rate of transition (probability per unit time) from a single state to set of states due to perturbation.

2. It provides the rate of stimulated emission and stimulated absorption.
3. The transition probabilities are known as decay probability.

If the transition takes place to state k of energy b/w E_K and $E_K + dE_K$. Let the energy density is $\rho(k)$ [~~energy~~
Then the transition probability per unit time is defined as

$$\omega = \frac{1}{t} \int |a_K^{(1)}(t)|^2 \rho(K)\, dE_K$$

where $\rho(K)$ are the no. of energy states b/w E_K and $E_K + dE_K$.

$$\omega = \frac{1}{t}\int_{-\infty}^{+\infty} \frac{4|\langle K|H'|n\rangle|^2 \sin^2(\omega_{Kn}t/2)}{\omega_{Kn}^2 \hbar^2} \rho(K)\, dE_K$$

$$= \frac{|\langle K|H'|n\rangle|^2 \rho(K)}{\hbar^2 t}\int_{-\infty}^{+\infty} \frac{4\sin^2(\omega_{Kn}t/2)\hbar\, d\omega_{Kn}}{\omega_{Kn}^2}$$

[Since $dE_K = E_K - E_n = \hbar\, d\omega_{Kn}$] Since t is large then the central peak is sharp and therefore $\rho(k)$ and $\langle k|H'|n\rangle$ may be independent of E_k so these may be taken out the integral

$$\omega = \frac{|\langle K|H'|n\rangle|^2 \rho(K)}{\hbar t}\int_{-\infty}^{+\infty} \frac{\sin^2(\omega_{Kn}t/2)\, d\omega_{Kn}}{(\omega_{Kn}/2)^2}$$

$$\omega = \frac{|\langle K|H'n\rangle|^2 \rho(K) . 2\pi t}{\hbar t}$$

$$\int_{-\infty}^{+\infty} \frac{\sin^2 \omega_{Kn}t/2}{(\omega_{Kn}/2)^2} d\omega_{Kn} = 2\pi t$$

$$\boxed{\omega = \frac{2\pi}{\hbar} \rho(K) |\langle K|H'|n\rangle|^2}$$

This is Fermi - Golden Rule.

From this rule it is clear that :-

1. 'ω' does not contain t, therefore, it is also known as constant perturbation.

2. $\omega \propto \rho(K)$.

3. $\omega \propto |\langle K|H'|n\rangle|^2$ i.e; momentum remains conserved.

4. ω is always positive.

There are many states $K_1, K_2, K_3, \ldots$ of the same state 'K'. The energy of these states $K_1, K_2, K_3, \ldots$ is almost same and hence, $\rho(K)$ and $\langle K|H'|n\rangle$ will be nearly constant for each group and will differ from one group to another.

E_k so they may be taken out the integral

$d\omega_{kn}$

Therefore, this relation of probability per unit time may be written as

$$\omega = \frac{2\pi}{\hbar} \rho(K_j) |\langle K_j|H'|n\rangle|^2$$

where $j = 1, 2, 3, \ldots$

In this rule both energy and momentum remains conserved.

* This rule is used to calculate the transistion probabilities b/w two states and their corresponding life times

* Fermi-Golden rule violets Time Dependent Perturbation theory.

* Inspite of this it agrees with Time Dependent Perturbation Theory.

* During transition the uncertainity principle plays its role. i.e;
$\Delta E \cdot \Delta t \simeq \hbar$

From this $\Delta E = \frac{\hbar}{t}$ and this agrees with the width of the peak.